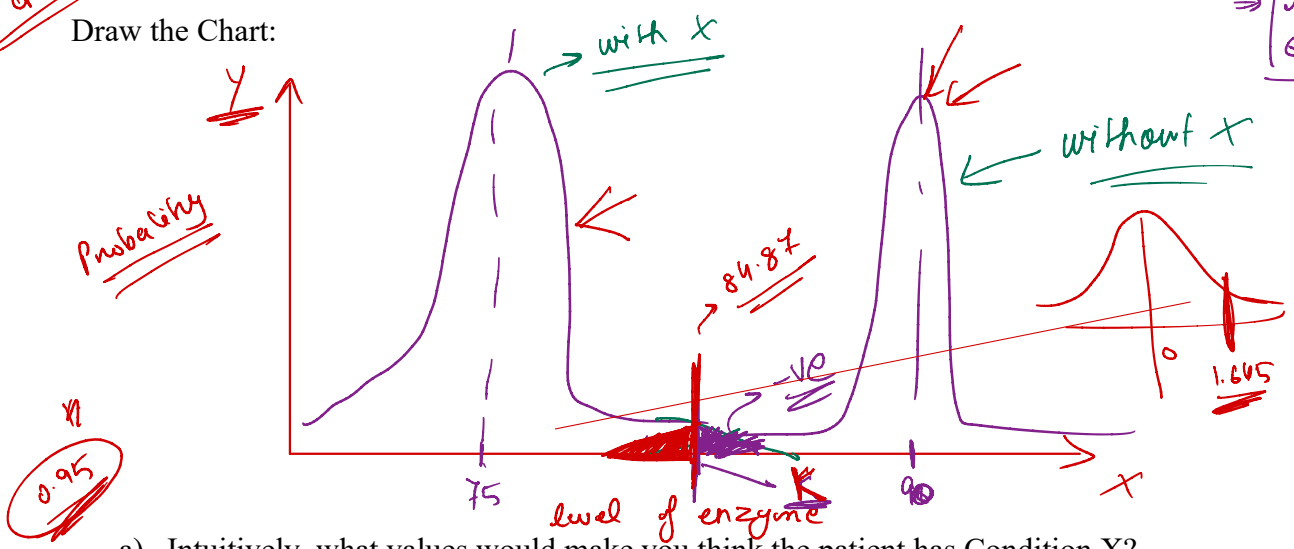
Example (Test Positive for Condition X):

For individuals with Condition X, the level of enzyme Y in the blood is normally distributed with a mean of 75 and a standard deviation of 6. For individuals without Condition X, enzyme Y levels are normally distributed with a mean of 90 and a standard deviation of 3.

Draw the Chart:



a) Intuitively, what values would make you think the patient has Condition X?

b) At what enzyme Y level should the "Tested Positive for Condition X" threshold start so that only 0.05 of people with Condition X would test negative?

$$\frac{X - \mu_0}{6} = 1.645$$

$$X = 1.645 \times 6 + 75 \Rightarrow 84.87$$

c) What would be the probability of a false positive (an individual without Condition X tests positive)?

$$\frac{84.87 - 90}{3} = \Phi(K) \qquad K = 0.9564$$

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d) A patient with Condition X has an enzyme Y level of 80. Will we properly diagnose that patient?

84.8%

Yes

e) A patient with Condition X has an enzyme Y level of 85. Will we properly diagnose that patient?

No

f) A healthy patient has an enzyme Y level of 80. Will we properly diagnose that patient?

No

g) A healthy patient has an enzyme Y level of 85. Will we properly diagnose that patient?

Yes

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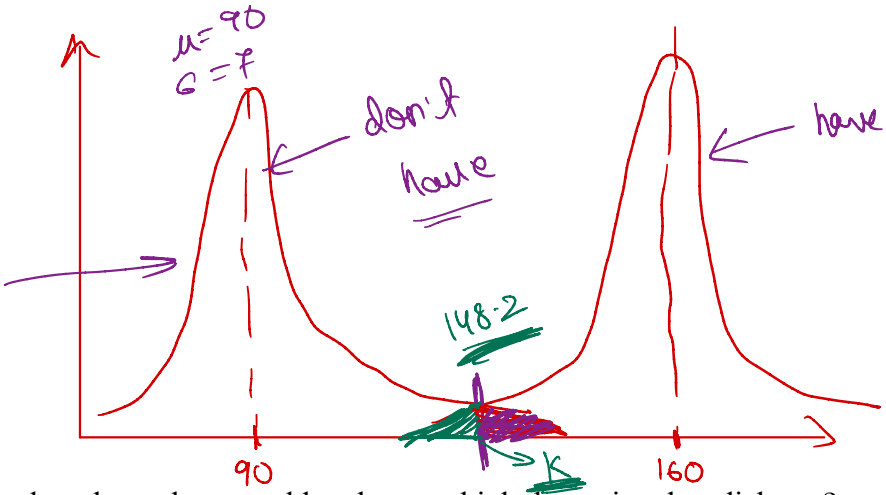
Example (Blood Sugar Levels): For individuals with diabetes, blood sugar levels (mg/dL) after fasting are normally distributed with a mean of 160 and a standard deviation of 10. For individuals without diabetes, the distribution has a mean of 90 and a standard deviation of 7.

Draw the Chart



148.2 > has
< not

a) Intuitively, what values would make you think the patient has diabetes?

b) Where would you set the threshold for "Tested Positive for Diabetes" so that the probability of a diabetic patient testing negative is ~~0.001~~? → 0.01 ?

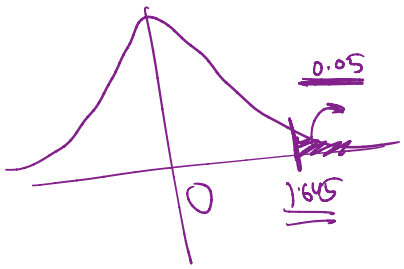
$K =$

$$\Rightarrow \frac{K-\mu}{\sigma} = \frac{K-160}{10} \qquad -1.28 = \frac{K-160}{10}$$

$$\Rightarrow \Phi\left(\frac{K-160}{10}\right) = 0.01 = \Phi(-1.28) \qquad \Rightarrow K = 160 - 12.8 = 148.2$$

c) What is the probability of a false positive (non-diabetic individual tests positive)?

$$\Rightarrow \frac{K-\mu}{\sigma} = \frac{148.2-90}{7} \approx 1.645$$



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d) A diabetic patient has a fasting blood sugar level of 150. Will we properly diagnose that patient?

Yes

e) A diabetic patient has a fasting blood sugar level of 170. Will we properly diagnose that patient?

Yes

f) A non-diabetic individual has a fasting blood sugar level of 150. Will we properly diagnose that patient?

No

g) A non-diabetic individual has a fasting blood sugar level of 170. Will we properly diagnose that patient?

No

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